## Active Galactic Nuclei and Super-Massive Black Holes

#### **1** Units to be used throughout this Lab:

Because different kinds of units are needed in this Lab Exercise, some conversions are listed below: 1 astronomical unit = 1 a.u. = average Earth to Sun distance =  $1.5 \times 10^8$  km 1 parsec = 1 pc =  $3.09 \times 10^{13}$  km = 206265 astronomical units = 3.26 light-year.

1 kiloparsec = 1 kpc = 1000 parsecs =  $10^3$  pc =  $3.09 \times 10^{16}$  km.

1 Mega-parsec = 1 Mpc =  $10^3$  kilo-parsecs =  $10^6$  parsecs =  $3.09 \times 10^{19}$  km.

#### 2 Introduction to Active Galactic Nuclei and Super-Massive Black Holes

A Black Hole (BH) is a region in space where the density is so large that the escape velocity needed to is larger than the speed of light c — Einstein's theory of Relativity states that nothing can travel faster than light, so that not even light can escape from a black hole — hence a BH is truly a dark object, almost completely isolated from the rest of the universe except for its very strong gravity. In the cosmos, BH exist of masses ranging from stellar masses a few times the mass of the Sun or  $M_{\odot}$  all the way up to Super-Massive Black Holes (SMBH's) with masses in the range  $10^{6}-10^{10} M_{\odot}$ . Studies in astrophysics have shown that SMBH's are plentiful in the Universe, and leave their dramatic imprints all over the cosmos (see Fig. 1).

Cosmic sources of radiation powered by SMBH's are brightest known objects in the whole Universe, and can therefore be seen literally out to the largest known distances (13 billion lightyears or 4000 Mpc), *i.e.*, to well within the first billion years after the Big Bang explosion that started the expansion of the Universe 13.7 billion years ago. Active Galactic Nuclei (AGN) are the very energetic central regions of galaxies, that produce tremendous amounts of electromagnetic radiation, due to either the processes *around* a SMBH or due to star-formation in the center of the galaxy. In most cases, AGN have both a SMBH in their center, which is surrounded by a burst of active star-formation that produces a lot of young hot stars which are embedded in large amounts of gas dust. This gas and dust surrounds the AGN in a donut-shaped torus, and in many cases obscures our view of the AGN, unless we view the dust torus from directly "above" or "below" (see Fig. 2).

Over the last four decades, thousands of astronomers world-wide have studied AGN and SMBH's. Because of AGN and SMBH are often obscured by dust, it has taken that long and that many efforts to understand what exactly is happening with these "central monsters" in galaxies. From careful statistical studies, we do now know that most massive galaxies with an old bulge of stars do host an SMBH. This SMBH typically has 0.5% of the total bulge mass of the galaxy. In a giant galaxy today, the central SMBH can have swallowed up to  $10^{10} M_{\odot}$  in mass over the history of the universe. We also know from statistical studies that the AGN is visibly active only about 1% of the time. On a human timescale, this corresponds to SMBH's eating their food for no longer than about 15 minutes a day – a veritable fast food diet. AGN are like fireflies in the sky, they are only visible as "active" when the SMBH is actually feeding on gas, dust and stars from the surrounding dust-torus.

Despite this sporadic feeding of the central monster and the fact that SMBH's comprise less than 1% of the total galaxy mass in the universe, SMBH's produce more than 95% of the total electromagnetic radiation in the Universe. AGN are thus highly special events, and their root cause — SMBH's — are the most dramatic and dominant objects in the whole universe. The

existence of Black Holes (BH's) was predicted by Einstein's theory of General Relativity almost a century ago, and the first convincing observations of BH's and SMBH's appeared in the 1970's.

This AGN and SMBH Lab is thus not a simple matter, but because of its tremendous importance in astrophysics — and its likely interest to you — we will give it a try this semester. Because of the complexity of the material, carefully reading this Lab constitutes an essential part of the exercise. This Lab is new as of 2006 and still under development, so please bear with us.

#### 3 What do we know about Active Galactic Nuclei and Super-Massive Black Holes?

Systematic studies over the last four decades have shown that there are three main types of AGN:

(a) Quasars, ("quasi-stellar objects" or QSOs), which are very compact objects that resemble stars in optical images;

(b) Seyfert galaxies, which appear to be normal spiral galaxies but with fluctuations in brightness in their centers; and

(c) Radio galaxies, which are mostly early type or elliptical galaxies, which emit massive jets of gas powered by SMBH's in their centers. The difference between quasars and radio galaxies turns out to be largely a question of viewing angle.

In general, AGN have six main observational properties:

(1) AGN are very compact in angular size, indicating the relatively small region over which the enormous energy is emitted. We will learn in this Lab that this region is in general no bigger than our solar system, or a few 100 AU in diameter at most, and in some cases only a few 10's of AU's across.

(2) AGN emit extremely large luminosities, many billions or trillions times brighter than our own Sun. The most energetic quasars can emit luminosities that outshine a whole galaxy  $10^4$  times, and so are brighter than  $10^{15}$  suns! The fact that this emission comes from a region no larger than our solar system (which has only one shining Sun!), make the nuclei of active galaxies a very bad place to live. [Exposing yourself to constant Sun-shine on Earth gives an average human being skin-cancer within 30 years. If you lived 1 AU from and AGN, you'd get skin-cancer within 1 micro-second or one-millionth of a second. ]

(3) AGN emit radiation over a very large range of wavelengths from radio to X-rays and even gamma-rays. This so-called "continuum emission" is due to very fast moving electrons emitted from the region just outside the SMBH. These electrons are accelerated along very strong magnetic fields that trail along with the SMBH. Due to the repeated acceleration, the speed of these electrons becomes relativistic, meaning that their velocities are almost the speed of light ( $v \leq c=300,000$  km s<sup>-1</sup>). According to theory, relativistic electrons will spiral around a magnetic field, and in the process loosing energy in the form of radiation that is called "synchrotron emission". This kind of radiation was first discovered in ground-based particle accelerators called "synchrotrons". This continuum or synchrotron emission released from the area immediately surrounding the SMBH makes the AGN. Note that by definition a BH is invisible, so it is only the brightly shining area immediately surrounding the SMBH that can be seen.

(4) AGN show very strong emission lines in their optical spectra, indicating the presence of atomic gasses that are highly excited by the strong AGN emission surrounding the SMBH. The Doppler wavelength shifts of these atomic emission lines suggest motions of the gas of many 1000's to 10,000's km s<sup>-1</sup>, which is much faster than the regular motion of stars and gas in the spiral disk of a galaxy. (Our Sun goes around the center of our own Galaxy at a typical speed of 220 km s<sup>-1</sup>). These super-fast motions in AGN are coming from a very small region, and indicate a tremendously violent environment of gas clouds immediately surrounding the SMBH, that are about to fall in. The SMBH acts like a giant kitchen garbage-disposal with rotation's at its "surface" close to the speed of light — imagine throwing in one trillion-trillion-million forks and knives into this garbage-disposal — half get forever sucked up by the SMBH, and the other half get thrown out — *before* they fall in — at nearly the speed of light (don't do this experiment at home). That's what constitutes an AGN!

(5) AGN have a fantastic direction for memory. The energy needed to produce the electromagnetic radiation from an AGN is generated by a SMBH, which sends out relativistic jets of high-energy particles many millions of light-years into intergalactic space. (Remember it is not the SMBH itself, but only the area immediately surrounding the SMBH which emits this radiation). These relativistic particles emit radio waves, which we can use to trace the jets and determine how much energy they contain. These objects can be seen as "radio galaxies", which emit highly collimated relativistic jets of radio waves from their central core. An example of an early-type galaxy emitting a radio jet is shown in Fig. 1. The relativistic jets start out at AU-scales (light-minutes to light-hours), then move to pc scales (several to many light-years). In spiral galaxies which have a dense interstellar medium, the jets get stopped by this surrounding medium, and so they only travel out to relative "small" distances of several pc. Spiral galaxies with such AGN jets are called "Sevfert" galaxies. Elliptical galaxies have a less dense interstellar medium, so that relativistic jets can continue up to kpc scales (thousands of light-years) or even Mpc scales (millions of light-years). The rapid rotation of the SMBH that stabilizes the jet to keep roughly the same direction over millions of years. This is illustrated in detail in Fig. 1–2. The inner jets of AGN have been mapped with the world's largest radio telescopes, and the smallest sizes that have been measured here provide a direct estimate of the inner accretion disk size and hence the SMBH size.

(6) Many AGN show time variability of their continuum and spectral line emission, which indicates the timescales at which gas and stars fall into the SMBH. Because of the rapid rotation of the SMBH, this infall occurs along an accretion disk, which is puffed up like a pc-sized donut orbiting the SMBH. The inner parts of the accretion disk are more highly flattened because of the rapid rotation of the SMBH, which leads to a DVD-shaped inner accretion disk that actually feeds the SMBH, as shown in more detail in Fig. 2. The inner accretion disk is many 10's of AU across, and whenever a gas-cloud or star falls into the SMBH, the area immediately surrounding the SMBH flares up with enormous amounts of synchrotron radiation, which we observe as the time-variable AGN. Because of the finite speed of light, the actual region causing the "flickering" AGN light cannot be larger than the distance light can travel in the typical AGN variability time-interval, which therefore provides a direct upper limit or estimate of the SMBH radius and mass.

# 4 Goal of this Lab:

Since General Relativity shows that the SMBH radius is directly proportional to its enclosed mass, we will us in this Lab methods (5) and (6) to directly estimate the SMBH sizes and masses for a number of AGN.

You can learn more about AGN and SMBH at www.nrao.edu/imagegallery/ , and also on the URL's: hubblesite.org/newscenter/newsdesk/archive/releases/category/galaxy (click on Quasar/Active Nucleus ), and on: hubblesite.org/news/2006/04 .



Fig. 1: VLA image of 0313-192: Optical image of an early-type galaxy (color; with enlargement shown in right panel) and of its extended radio source observed with the Very Large Array (red; with total extend shown in left panel). Note how the relativistic jets emanated from the galaxy center, and press well past the interstellar medium of this galaxy on both sides into intergalactic space, where they eventually get stopped and loose most of their energy. If this object were viewed face-on or "down-the-pipe", the relativistic amplification of the jet would turn the core of this galaxy into a quasar, which would greatly outshine the galaxy at all wavelengths.

Figure courtesy of W. C. Keel (U. Alabama), NASA and hubblesite.org/news/2003/04/ .



Fig. 2: Summary of how AGN affect their surroundings over 12 orders of magnitude in size: from AU scales (or  $\mu$ pc; lower right) to Mpc scales (300 Mlyr; upper left), or from General Relativistic singularity (AU-scales) to Relativistic Jets (Mpc-scales). Starting from the upper left, each next panel is expanded by a factor of 10. The SMBH is well visible in the lower right two panels, and the inner accretion disk and torus in the right 6 panels (AU-pc scales). The tremendous AGN radiation and relativistic jet are well visible in the left 6 panels (pc-Mpc scales), with the galaxy itself shown in the 100-kpc panel (2nd from upper left).

If the relativistic jet shines in your direction, then you see a quasar with a luminosity  $10^{15} L_{\odot}$  coming from an area as small as only several AU across! If you lived as close to an AGN as we live to the Sun (lower right 2 panels), your sky would be  $10^{15} \times$  brighter than it is in our solar system, *i.e.*, not a good place to live. AGN are so bright that they literally can be seen to the edge of the universe, or about 13.5 billion light-years away. Even though SMBH comprise less than 1% of the total mass in galaxies, they produce  $\gtrsim 95\%$  of all electromagnetic radiation in the universe.

Figure from R. Blandford in "Active Galactic Nuclei", 1990, Springer Verlag (Berlin; ISBN 3-540-53285-4).



Fig. 3: The Very Large Array (VLA) radio image of the radio galaxy M87, which is a giant elliptical galaxy in the center of the Virgo cluster of galaxies about 18 Mpc away. The top left image show giant, bubble-like structures where radio emission is thought to be powered by the jets from the galaxy's central SMBH. The middle left image shows the jets of subatomic particles coming from the core (with a Hubble Space Telescope HST image for comparison). The bottom left image shows the Very Long Baseline Array (VLBA) image of the region closest to the core, where the jet is formed into a narrow beam. White bars indicate the scales in each image: 10 kpc (kilo-parsecs) is equal to 32,600 light-years; 1 kpc equals 3,260 light-years; and 0.01 pc equals 0.0326 light- years, or 2,062 times the distance from the Earth to the Sun. The image to the right gives the large scale overview.

M87's jet formed within a few tenths of a light-year of the galaxy's core, presumed to be a SMBH three billion times more massive than the sun. In this region, the jet is seen opening widely, at an angle of about 60° closest to the SMBH, but is squeezed down to only 6° a few light-years away. At the center of M87, material being drawn inward by the strong gravitation of the SMBH is formed into a rapidly-spinning flat accretion disk. The subatomic particles are pushed outward from the poles of this disk. Scientists believe that magnetic fields in the disk are twisted tightly as the disk spins, and then channel the electrically-charged particles into a pair of narrow jets.

Figure courtesy of F. Owen, J. Biretta, J. Eilek & N. Kassim, and NRAO, and can also be found on: www.nrao.edu/imagegallery/php/level2.php?class=AGN



Fig. 4: The Very Large Array (VLA) radio image of the radio galaxy M87, which is one of the strongest radio sources in the sky. The different panels show repeated enlargements of the radio source over a factor of 30,000 in size. Its radio structure has been studied with ground-based radio telescopes on scales ranging from 50 kpc (50,000 light years; (the largest extent of the source) down to 0.08 pc (1/4 light year, about 100 times larger than the central SMBH).

Details can be found on: www.aoc.nrao.edu/~fowen/M87.html



Fig. 5: Very Long Baseline Image (VLBI) image of M87 at a redshift z=0.0044, made by the largest available ground-based radio interferometry network consisting of the VLA (35 km baseline), the continental VLBA (5000 km baseline), and the orbiting Japanese Halca satellite (VSOP; 76,000 km maximum baseline). At the highest resolution, the very inner part of M87's relativistic jet is shown with white contours. The putative SMBH in the center of M87 is at coordinates (X,Y)=(0,0). The orbiting satellite results in the radio resolution (or radio "seeing") not being round, but highlyelongated like the single white ellipses shows in the image, which are faint radio source components. The resolution of the image is  $2.3 \times 0.2$  m.a.s., where 1 m.a.s. = 1 milli-arcsecond = 0.001''. Each tick-mark along the axes is 1 m.a.s. Note how well the inner jet is aligned with the outer parts of the M87 jet in Fig. 3 and 4, illustrating the fantastic memory for direction that AGN have.

Image courtesy of H. Hirabayashi et al. 2000, Publications of the Astronomical Society of Japan, 52, 997 "The VSOP 5 GHz AGN Survey I. Compilation and Observations". Details can be found on: www.vsop.isas.jaxa.jp/survey/publications/

#### 5 Estimating SMBH masses from the smallest observed radio size

In this section we will measure the smallest observed radio size of a radio source surrounding an SMBH, and from there the SMBH mass. This was done with a network of ground-based radio interferometry dishes in Fig. 5, including an orbiting radio dish on the Japanese satellite "Halca" (also known as VSOP), which orbited up to 38,000 km away from Earth.

First, the wavelength and frequency of the observation are related as following:

(5a)  $\lambda = c / \nu$ 

where c = speed of light = 300,000 km s<sup>-1</sup> and the frequency of the radio observation is  $\nu = 5$  GHz. From this, I calculate that the wavelength of the radio observation is:

(5a) 
$$\lambda =$$
\_\_\_\_\_ cm.

Next, the resolution  $\theta$  of any telescope is defined as the smallest detail or angle it can see, and the typical resolution of a telescope — including a radio interferometer — is given by:

(**5b**)  $\theta_1 = 1.22 \times 206265 \times \lambda / B$  [in arcsec]

where  $\lambda$  is the wavelength of the radio observations and B is the maximum diameter (or Baseline B) of the telescope (or radio interferometer) used during the observations, all converted to the *same* units length. Note B is twice the maximum distance of the satellite from Earth! Hence, I calculate that the resolution R of the radio observation is:

(5b)  $\theta_1 =$ \_\_\_\_\_\_ arcsec = \_\_\_\_\_\_ milli-arcsec (m.a.s.)

I confirm this from Fig. 5, where I measure the radio blob in at (X,Y)=(0,0) in its smallest direction to have the following angular diameter:

(5b)  $\theta_2 =$ \_\_\_\_\_\_ tick-marks across = \_\_\_\_\_\_ (m.a.s.).

Note that the measurements are not perfectly consistent, so I will just take the average:

(**5b**)  $\theta = (\theta_1 + \theta_2) / 2$  (m.a.s.) = \_\_\_\_\_ (m.a.s.)

The SMBH cannot be bigger than  $\theta$  in angle on the sky. We now need to convert this angle with the known distance of the galaxy M87 to a physical size of the area immediately surrounding the SMBH, whose size was measured with this radio observation. For this we need to know the cosmological redshift z of the galaxy due to the expansion of the Universe, which is a direct measure of its distance D given Hubble's law:

(5c) 
$$D = (c/H_0) \times z$$
 (Mpc)

where c is again the speed of light, and the expansion rate of the Universe is indicated by the Hubble Constant  $= 71 \ km \ s^{-1} \ Mpc^{-1}$  (see the Hubble expansion Lab at the end of the semester). The Hubble redshift of M87 is z = 0.0044. From this I calculate the following distance to M87:

(5c) 
$$D =$$
\_\_\_\_\_  $(Mpc) =$ \_\_\_\_\_ pc.

Now the linear diameter d of the region of the radio emission surrounding the SMBH as viewed at distance D simply follow from the small angle approximation:

(5d) 
$$d = \theta \times D / 206265$$
 (in pc)

where  $\theta$  is the smallest angle measured above (in arcsec) for the radio blob directly on top of the suspected SMBH, and D is the distance in pc. From this, the diameter d I derive for this region is:

We know that the size of this region must be a bit bigger than the actual size of the SMBH, otherwise the radio emission could not have escaped. How much bigger we do not know, but from the model cartoons in the bottom right panels of Fig. 2 we estimate that the SMBH radius  $r_s$  could be a few  $\times -10 \times$  smaller (let's assume 5  $\times$  smaller) than the linear diameter d of the radio source:

(5e)  $r_s = d / 5 =$ \_\_\_\_\_/ 5 AU.

Our last step is to use the relation of the Schwarzschild radius of the SMBH to convert d into a Mass. The Schwarzschild is essentially the same as what we get for Kepler's third law of motion, except that in the relativistic case there is an extra factor of 2 in the equation:

(5f)  $M_{smbh} = r_s \times c^2 / (2 \times G)$  (gr)

where G = the gravitational constant =  $6.67 \times 10^{-8}$  (cgs), and c is the speed of light =  $2.998 \times 10^{10}$ (cm/sec). For this formula to work, all units must be in these cgs units, which are cm for length, cm/sec for speed, and grams or gr for mass). So I compute the following SMBH mass for M87 in grams:

(5f)  $M_{smbh} =$ \_\_\_\_\_(gr)

Since the mass of the Sun is  $M_{\odot} = 1.99 \times 10^{33}$  (gr), the SMBH mass measured in solar masses follows then as:

(5g) SMBH Mass =  $M_{smbh} / M_{\odot} =$ \_\_\_\_\_ (  $M_{\odot}$  ).

Please take the logarithm of this mass, enter it at the bottom of Table 2 below, and compare it to the literature value for the SMBH mass in M87, which is  $M_{smbh} = 3 \times 10^9 M_{\odot}$ :

(5g) Percentage error on  $M_{smbh}$  is:

Note that such estimates are uncertain (after all, we can't see what we measure), and any measurement in astrophysics like these are considered a success if they are within a factor of 3-10!

(5h) Please discuss the uncertainties in this diameter-based method. What would be required to improve upon this  $M_{smbh}$  estimate?



Fig. 6a. Light curve of the active galaxy Mrk 421 that has a Super-Massive Black Hole in its center.

Time-span $\Delta t$						
(hr)	(mag)	(AU)	$(M_{\odot})$			
Max1–Min1:						
Min1 More.						
Min1–Max2:						
Max2–Min2:						
Min2–Max3:						
Max3–Min3:						
Average:						

Table 1a. Analysis of the light curve of Mrk 421



Fig. 6b. Light curve of the active galaxy Mrk 501 that has a Super-Massive Black Hole in its center.

Time-span $\Delta t$	Approx. $\Delta$ mag	BH radius	BM mass
(hr)	(mag)	(AU)	$(M_{\odot})$
Max1–Min1:			
Min1–Max2:			
Max2–Min2:			<u> </u>
Min2–Max3:			
Max3–Min3:			
	<u> </u>		
Average:			

Table 1b. Analysis of the light curve of Mrk 501



Fig. 6c. Light curve of the radio galaxy OJ 287 that has a Super-Massive Black Hole in its center.

Time-span $\Delta t$			
(hr)	(mag)	(AU)	$(M_{\odot})$
Max1–Min1:			
Min1–Max2:			
Max2–Min2:			
Min2–Max3:			
Max3–Min3:			
Average:			
<u> </u>			

Table 1c.	Analysis	of the	light	curve	of OJ	287
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Fig. 6d. Light curve of the radio galaxy Perseus A that has a Super-Massive Black Hole in its center.

$\begin{array}{c} \text{Time-span } \Delta t \\ \text{(hr)} \end{array}$	$\frac{1}{\text{Approx. } \Delta \text{ mag}}$ (mag)	
Max1–Min1:		 
Min1–Max2:		 
Max2–Min2:		 
Min2–Max3:		 
Max3–Min3:		 
·		 
Average:		 

Table 1d. Analysis of the light curve of Perseus A



Fig. 6e. Light curve of the active galaxy S50716+71 that has a Super-Massive Black Hole in its center.

$\begin{array}{c} \text{Time-span } \Delta t \\ \text{(hr)} \end{array}$		BM mass ( $M_{\odot}$ )
Max1–Min1:	 	
Min1–Max2:	 	
Max2–Min2:	 	
Min2–Max3:	 	
Max3–Min3:	 	
Average:	 	

Table 1e. Analysis of the light curve of S50716+71

### 6 Estimating SMBH masses from the typical AGN variability time-scale

In this section we will estimate the SMBH mass  $(M_{smbh})$  from the variability seen in the light curves of up to five AGN measured with the ASU Braeside telescope near Flagstaff. The TA will tell you how many of the 5 AGN in Fig. 6a–6e you will be measuring. In good weather conditions, one of the TA's may also take some "live" observations with the ASU Braeside telescope of one of the variable AGN that is visible during class night.

To estimate SMBH masses from the typical AGN variability time-scales we will proceed as following. The typical time-span by which each AGN is referred to as the "variability period"  $\Delta t$ , even though the AGN feeding is not strictly a periodic phenomenon, unlike the case of variable stars which usually vary in a regular manner.

The SMBH mass can now be derived from the average time-span values  $\Delta t$  in Tables 1a–1e. Here we use the fact that — because of the finite speed of light — the actual region causing the "flickering" AGN light cannot be larger than the distance light can travel in the variability time-interval  $\Delta t$ . The diameter d of the variability region can thus not be larger than:

(6a)  $d = c \times \Delta t$  (cm).

The Schwarzschild of the SMBH must be less than half of this diameter, or:

(6b)  $r_s = 0.5 \times d = 0.5 \times c \times \Delta t$  (cm).

We use again the Schwarzschild equation above to convert our estimate of  $r_s$  into a direct estimate of (or upper limit to) the SMBH mass:

(6c)  $M_{smbh} = r_s \times c^2 / (2 \times G)$  or (6c)  $M_{smbh} = 0.5 \times c^3 \times \Delta t / (2 \times G)$  (gr).

Again, the gravitational constant  $G = 6.67 \times 10^{-8}$  (cgs), and the speed of light  $c = 2.998 \times 10^{10}$  (cm/sec). Since the mass of the Sun is  $M_{\odot} = 1.99 \times 10^{33}$  (gr), the SMBH mass measured in solar masses is then:

(6d) Mass =  $M_{smbh}$  /  $M_{\odot}$ 

Please code up equations (6a)–6d) in your calculator, and evaluate your estimate of  $M_{smbh}$  for each of the assigned AGN. Take again the logarithm of these masses, enter them in Table 2 below, and compare them to the literature values in Table 2, using:

(6d) Percentage error on  $M_{smbh}$  is: 100% × (Your-value — Literature-value) / (Literature-value)

(6e) Please discuss the uncertainties in this variability method. What are the inherent uncertainties and what would be required to improve upon this  $M_{smbh}$  estimate? Why do you think this method provides a more accurate SMBH mass than the diameter method in §5?

AGN name	Your derived $\log(M_{smbh})$ ( $M_{\odot}$ )	Published $\log(M_{smbh})$ ( $M_{\odot}$ )	Publ. error in $\log(M_{smbh})$	Redshift	Percentage error in $M_{smbh}$
Mrk421		8.50	0.18	0.030	
Mrk501		8.93	0.21	0.033	
PerA		8.53	0.18	0.018	
OJ287		8.60	0.20	0.310	
S50716+71		9.0	?	0.30?	
M87		9.5	0.3	0.0044	
Average $M_{smbh}$		8.84	0.4 (range)		

Table 2. Super-Massive Black Hole masses from AGN variability studies

 $({\bf 6f})$  Discuss in general what you learned from this Lab: